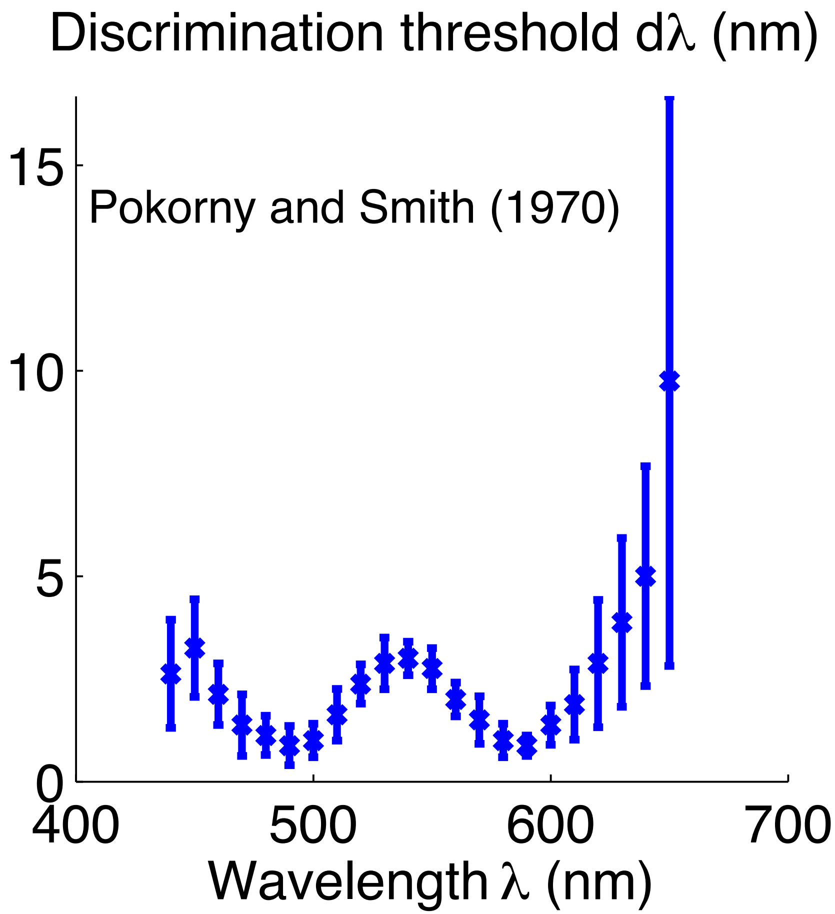
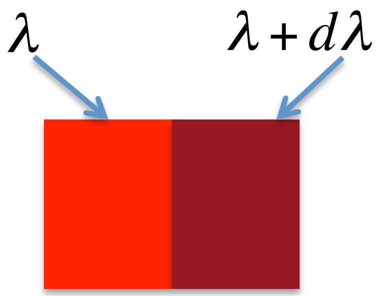
Understanding behavioral thresholds in discriminating the wavelength of monochromatic light from properties of cones and retina

Li Zhaoping,   at the Kongsberg Vision Meeting, Oct. 27, 2014

Based on Zhaoping, Geisler, and May, 2011.

Monochromatic discrimination threshold depends on wavelength (Pokorny and Smith, 1970)

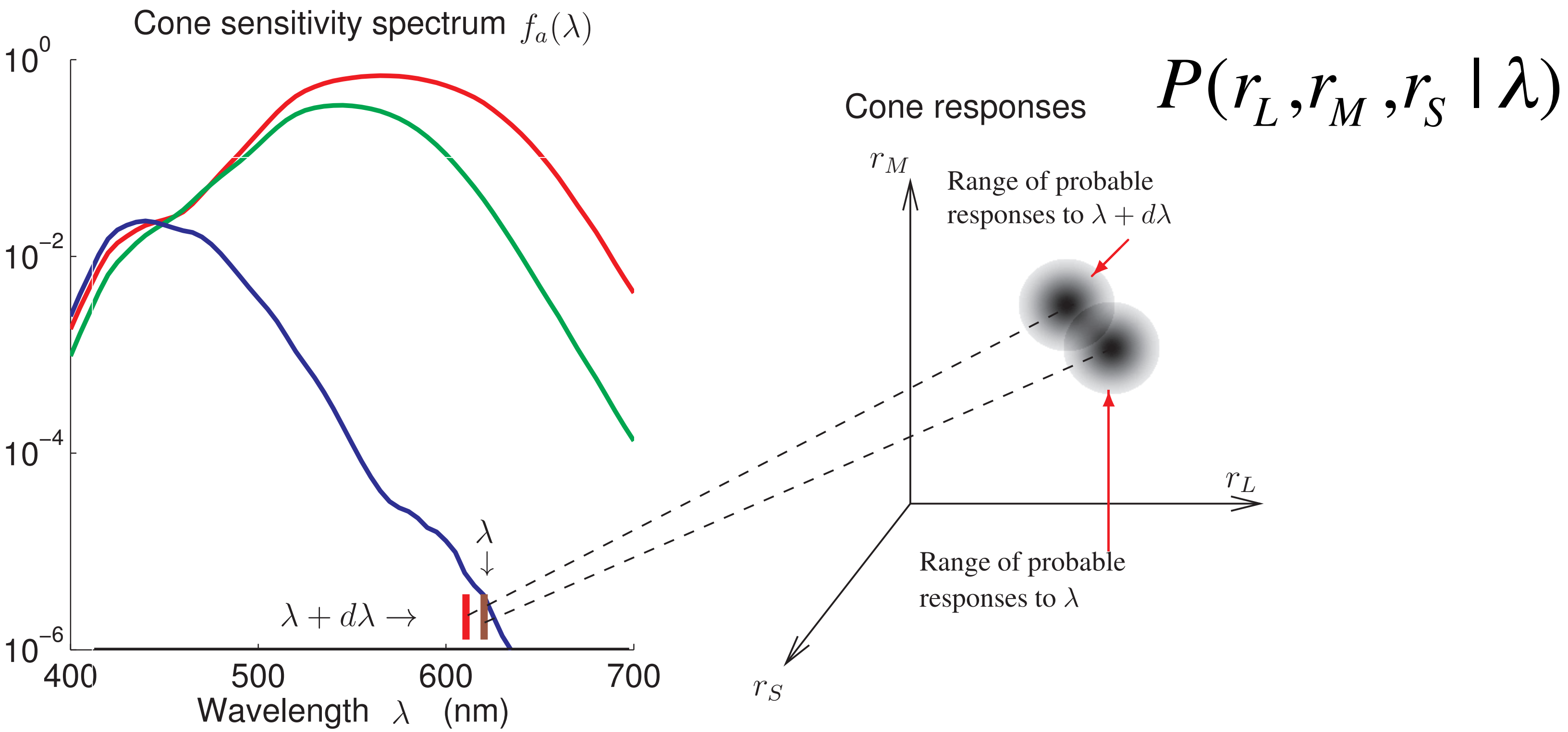




Subject adjust the intensity of the test field to make it appear identical to the standard field, threshold is reached when this is impossible

Why? Can we understand this from the information in the cone absorptions regardless of the post-receptor mechanisms (like an ideal observer's approach)

The basic gist: from cone response noise to discrimination threshold



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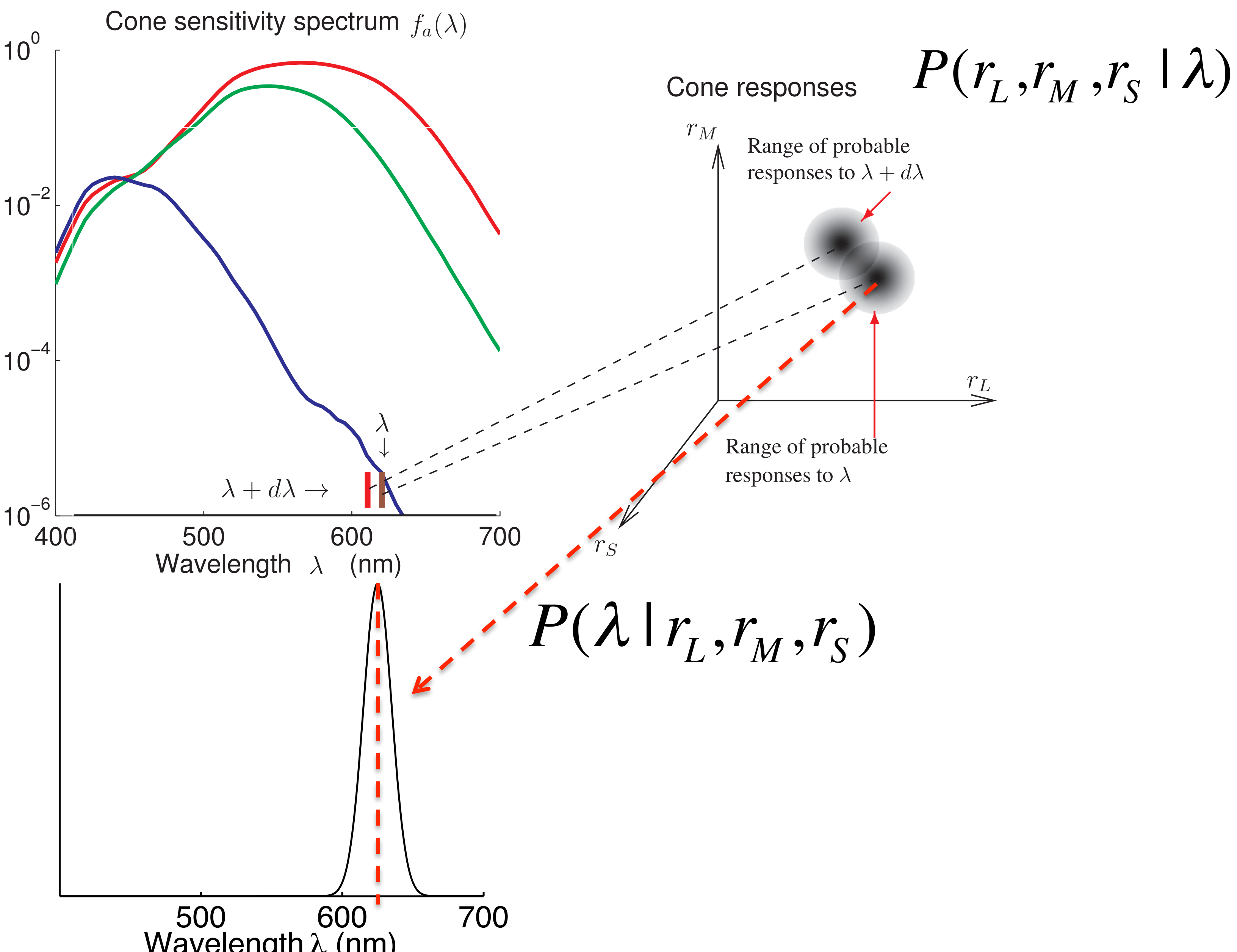
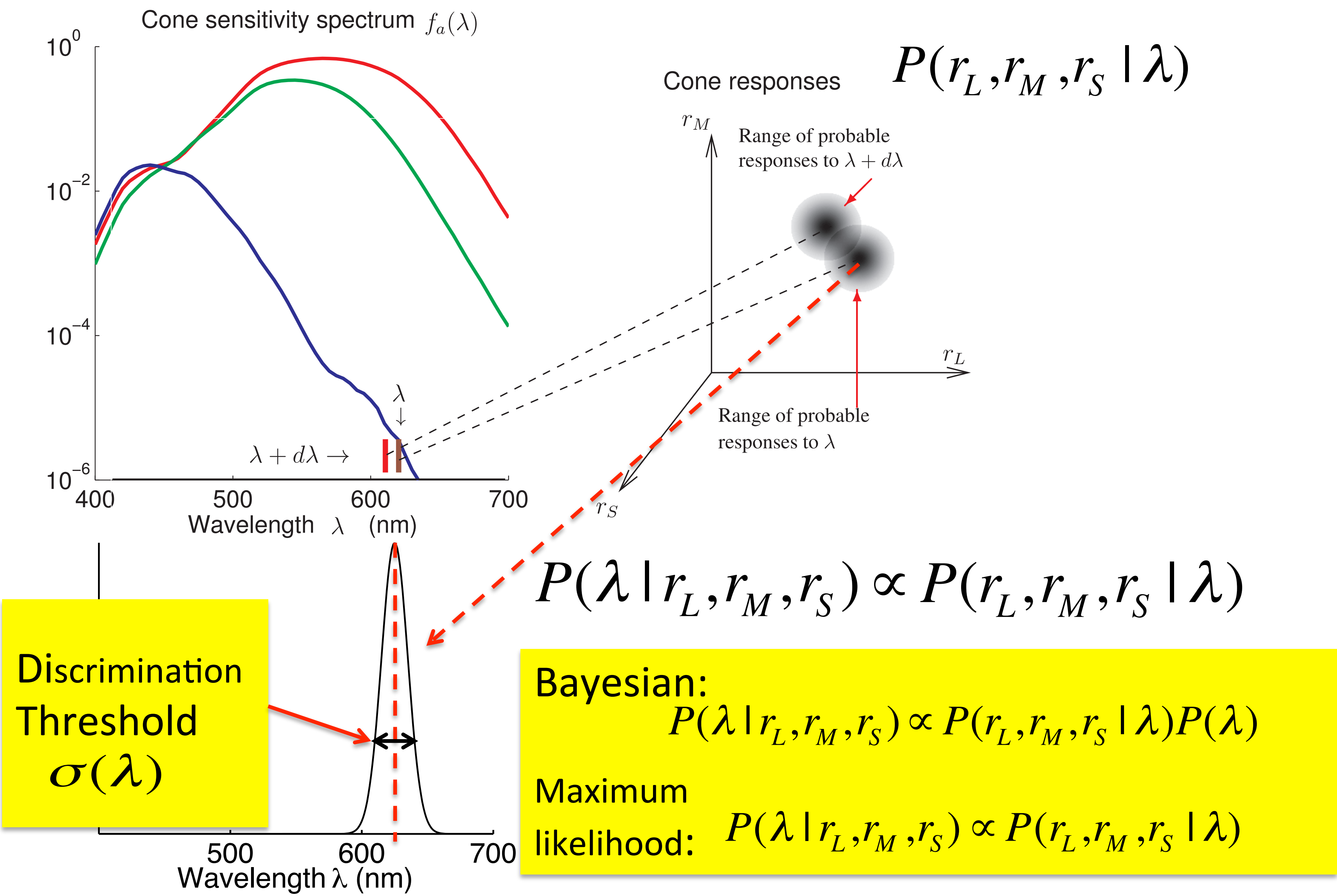
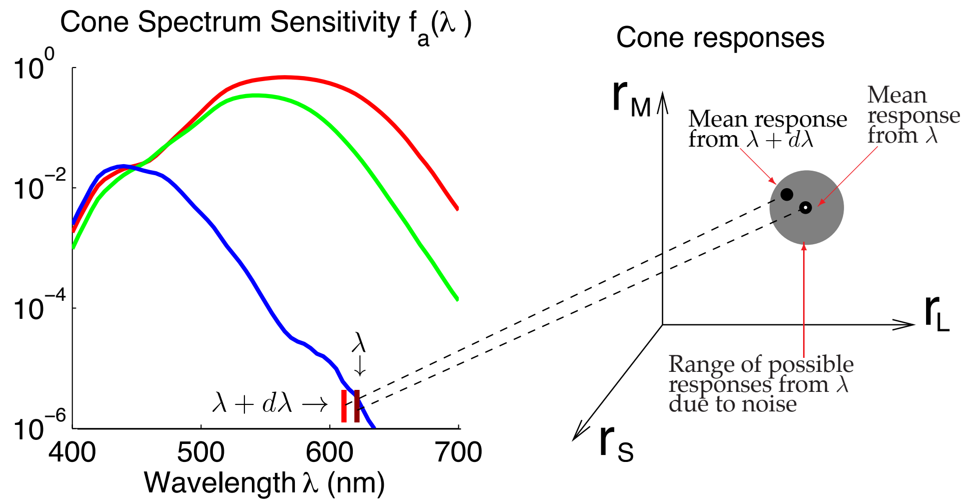


The basic gist: from cone response noise to discrimination threshold



First let us obtain $P(r_L, r_M, r_S \mid \lambda)$



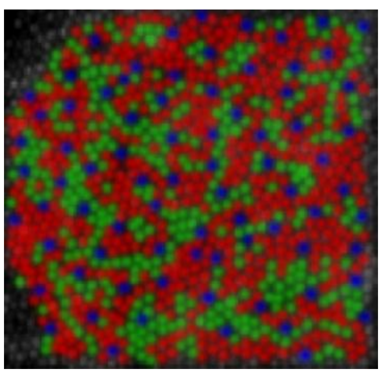
Mean cone response

$$\bar{r}_a = I \bullet f_a(\lambda)$$

Input light intensity

Spectral sensitivity

Number of cones n_L, n_M, n_S



$$n_L f_L(\lambda) \rightarrow f_L(\lambda)$$

$$n_M f_M(\lambda) \rightarrow f_M(\lambda)$$

$$n_S f_S(\lambda) \rightarrow f_S(\lambda)$$

$$n_L : n_M : n_S = 6:3:1$$

Pre-receptor absorption: O_L, O_M, O_S

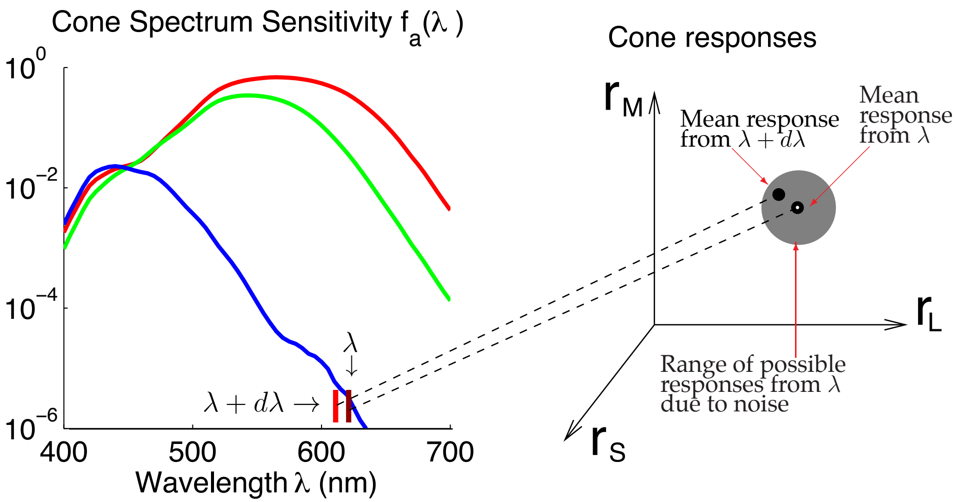
$$O_L f_L(\lambda) \rightarrow f_L(\lambda)$$

$$O_M f_M(\lambda) \rightarrow f_M(\lambda)$$

$$O_S f_S(\lambda) \rightarrow f_S(\lambda)$$

$$O_L : O_M : O_S = 1:1:0.2$$

First let us obtain $P(r_L, r_M, r_S \mid \lambda)$



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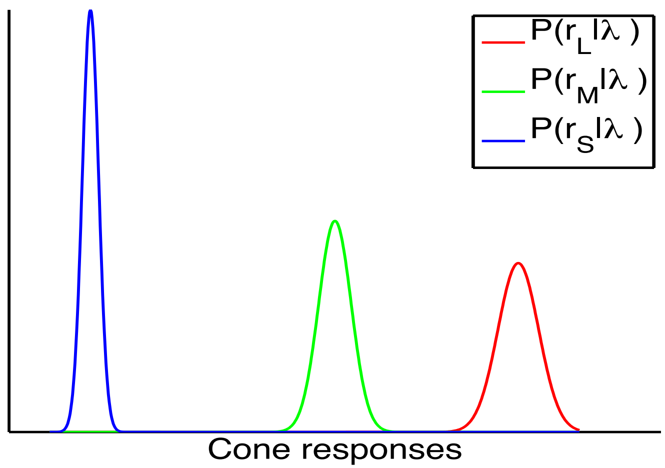
Input light intensity

Spectral sensitivity



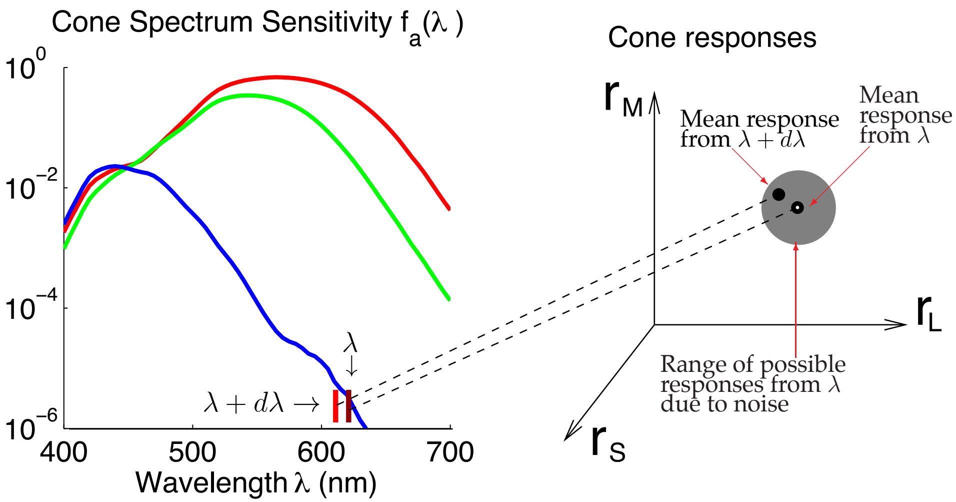
Cone response noise is Poisson:

$$P(r_a \mid \lambda) = \frac{\bar{r}_a^{\,r_a}}{r_a!} \exp(-\bar{r}_a)$$

Response noise in different cones are independent:

$$P(r_L, r_M, r_S \mid \lambda) = P(r_L \mid \lambda) P(r_M \mid \lambda) P(r_S \mid \lambda)$$

Cone response noise is Poisson noise $P(r_L, r_M, r_S \mid \lambda)$



Mean cone response

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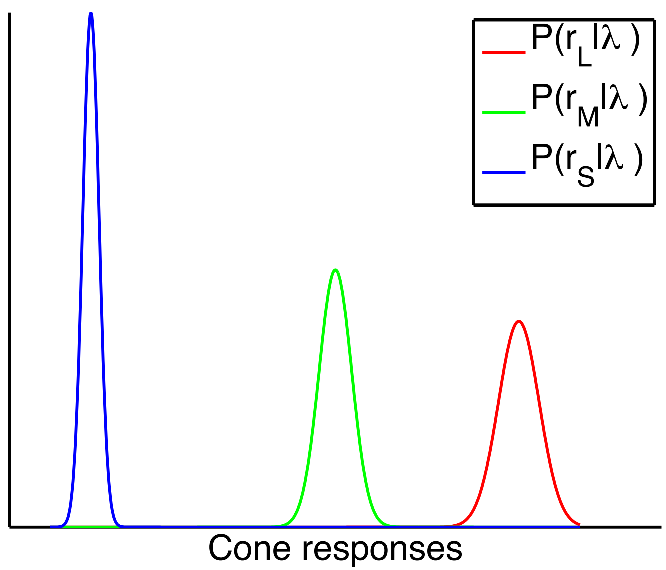


$$P(r_a \mid \lambda) = \frac{\bar{r}_a^{\,r_a}}{r_a!}\exp(-\bar{r}_a)$$

$$P(r_L, r_M, r_S \mid \lambda) = P(r_L \mid \lambda)P(r_M \mid \lambda)P(r_S \mid \lambda)$$

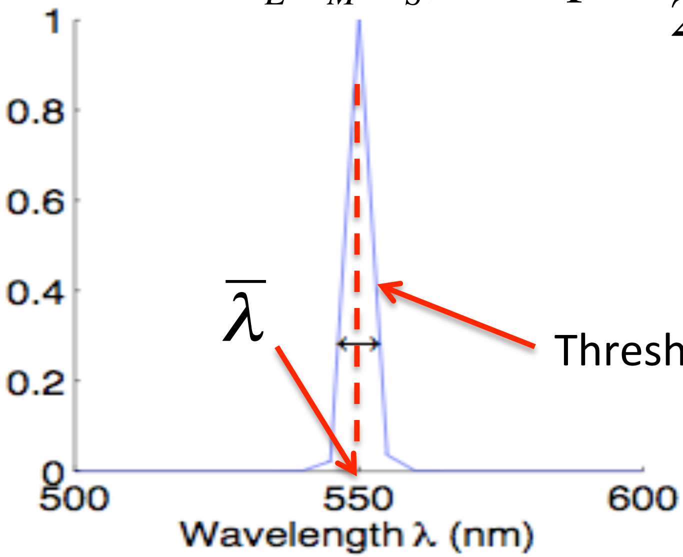
Next: $P(\lambda | r_L, r_M, r_S) \propto P(r_L, r_M, r_S | \lambda)$

An example of maximum likelihood decoding, given (r_L, r_M, r_S)

$\bar{r}_a = I \bullet f_a(\lambda)$

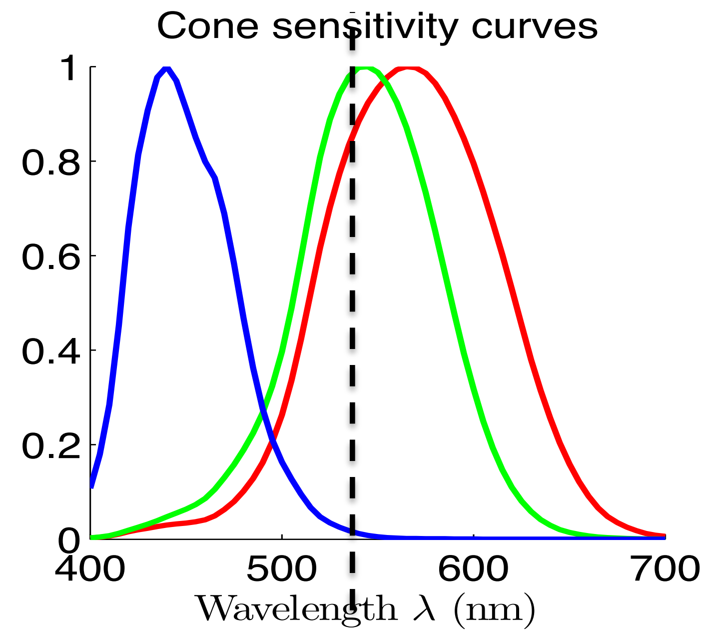
Responses generated by wavelength at 550 nm

$$P(\lambda | r_L, r_M, r_S) \approx \exp[-\frac{(\lambda - \bar{\lambda})^2}{2\sigma^2(\bar{\lambda})}]$$



$\bar{\lambda}$

Threshold value $\sigma(\bar{\lambda})$



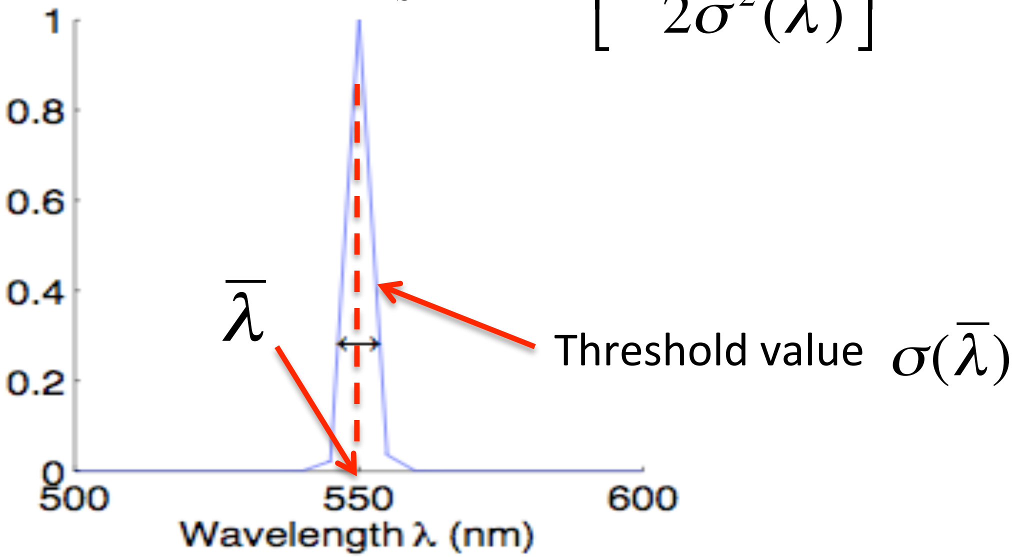
$$\text{Fisher Information } I_F(\lambda) = \sum_a I[f'_a(\lambda)]^2 / f_a(\lambda)$$

$$\text{threshold } \sigma(\lambda) = I_F^{-1/2}$$

Next: $P(\lambda | r_L, r_M, r_S) \propto P(r_L, r_M, r_S | \lambda)$

$$\ln P(\lambda | r_L, r_M, r_S) = \ln P(r_L, r_M, r_S | \lambda)$$
$$= \ln P(r_L | \lambda) + \ln P(r_M | \lambda) + \ln P(r_S | \lambda)$$

$$P(\lambda | r_L, r_M, r_S) \approx \exp\left[-\frac{(\lambda - \bar{\lambda})^2}{2\sigma^2(\bar{\lambda})}\right]$$



$$P(r_a | \lambda) = \frac{\bar{r}_a^{r_a}}{r_a!} \exp(-\bar{r}_a)$$

$$\bar{r}_a = I \bullet f_a(\lambda)$$

$$\frac{\partial \ln P(\lambda | r_L, r_M, r_S)}{\partial \lambda} = 0 \rightarrow \bar{\lambda}$$

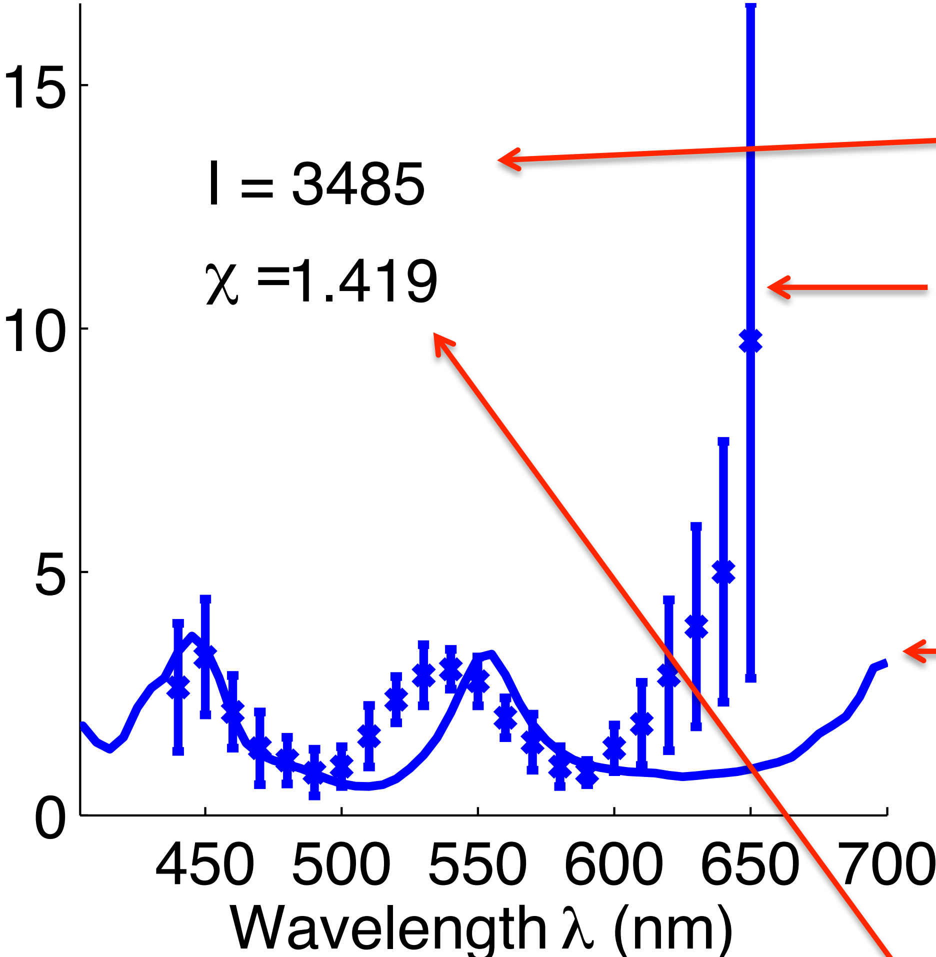
$$\frac{\partial^2 \ln P(\lambda | r_L, r_M, r_S)}{\partial \lambda^2} \rightarrow \sigma(\lambda)$$

Fisher Information $I_F(\lambda) = -\left\langle \frac{\partial^2 \ln P(\lambda | r_L, r_M, r_S)}{\partial \lambda^2} \right\rangle = \sum_a I[f'_a(\lambda)]^2 / f_a(\lambda)$

threshold $\sigma(\lambda) = I_F^{-1/2}$

Repeat this for all wavelength to find the wavelength discrimination threshold

$$\sigma(\lambda) = \{\sum_a I[f'_a(\lambda)]^2 / f_a(\lambda)\}^{-1/2}$$



chosen to minimize fitting error χ

Data $\sigma_{\text{data}}(\lambda)$

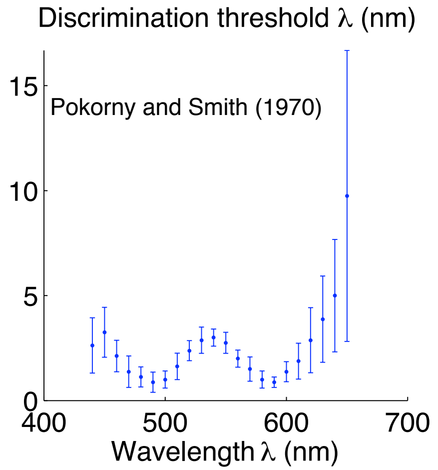
Why large discrepancy χ ?

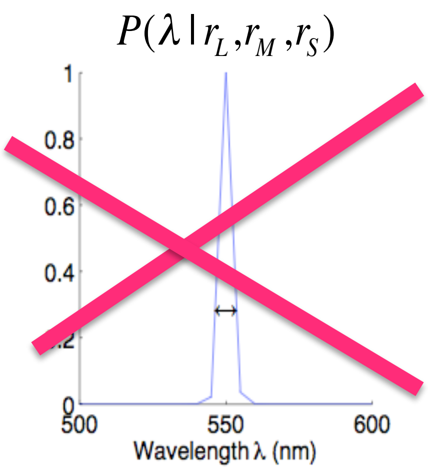
Model prediction $\sigma_{\text{model}}(\lambda)$

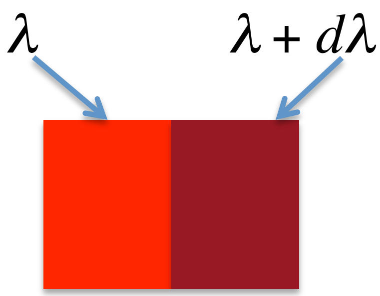
$$\chi^2 = \sum_\lambda \frac{[\sigma_{\text{model}}(\lambda) - \sigma_{\text{data}}(\lambda)]^2}{[\Delta\sigma_{\text{data}}(\lambda)]^2} \Bigg/ \sum_\lambda 1$$

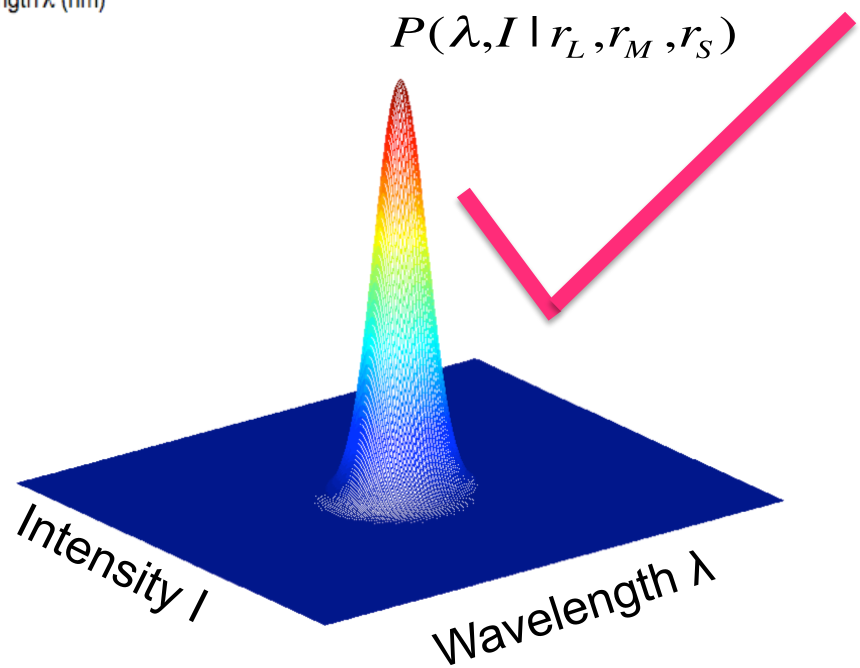
Back to the original measurements (Pokorny and Smith, 1970)

Both input intensity I and input wavelength λ are changed in matching.









Subject adjust the test field by intensity I to make it appear identical to the standard field, threshold is reached when this is impossible

Back to the original measurements (Pokorny and Smith, 1970)

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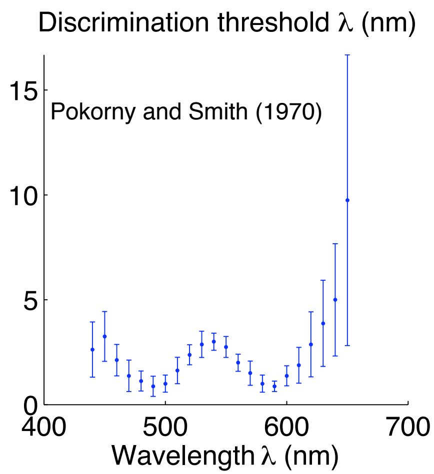
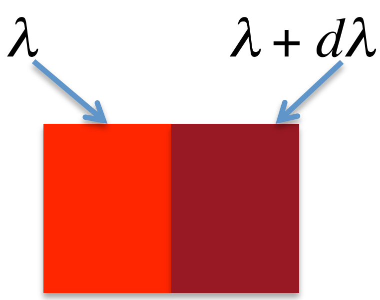
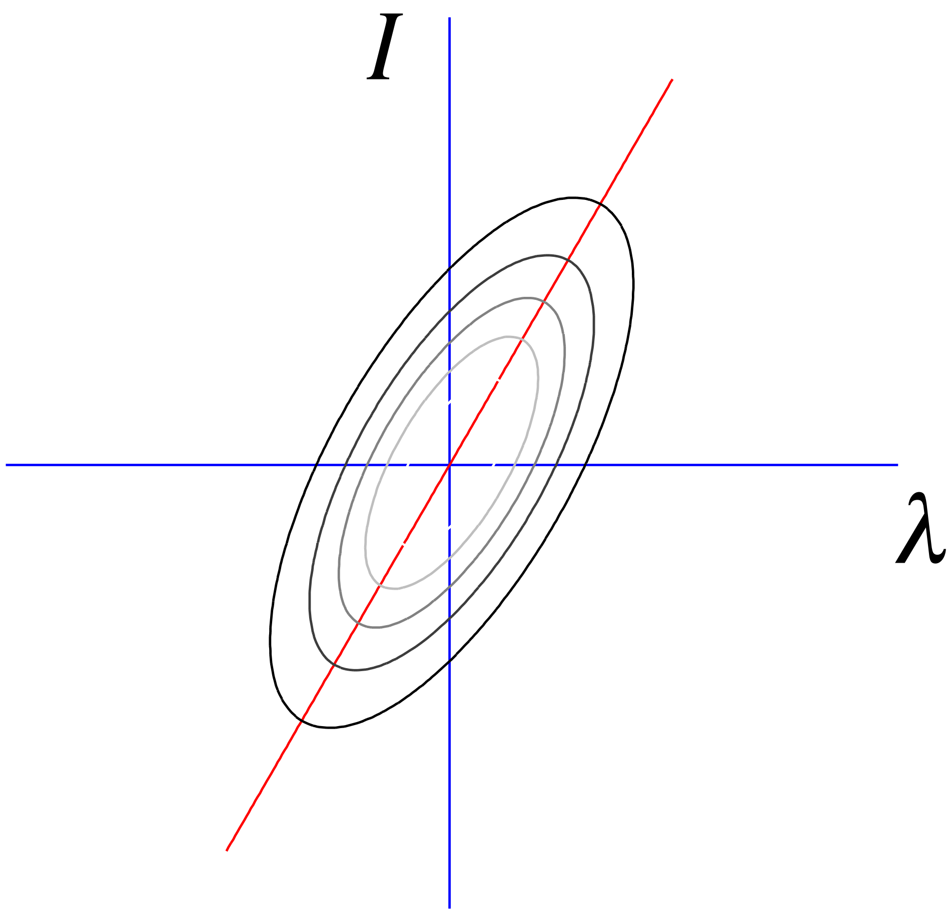




Subject adjust the test field by intensity I to make it appear identical to the standard field, threshold is reached when this is impossible

Contour plot $P(\lambda, I \mid r_L, r_M, r_S)$



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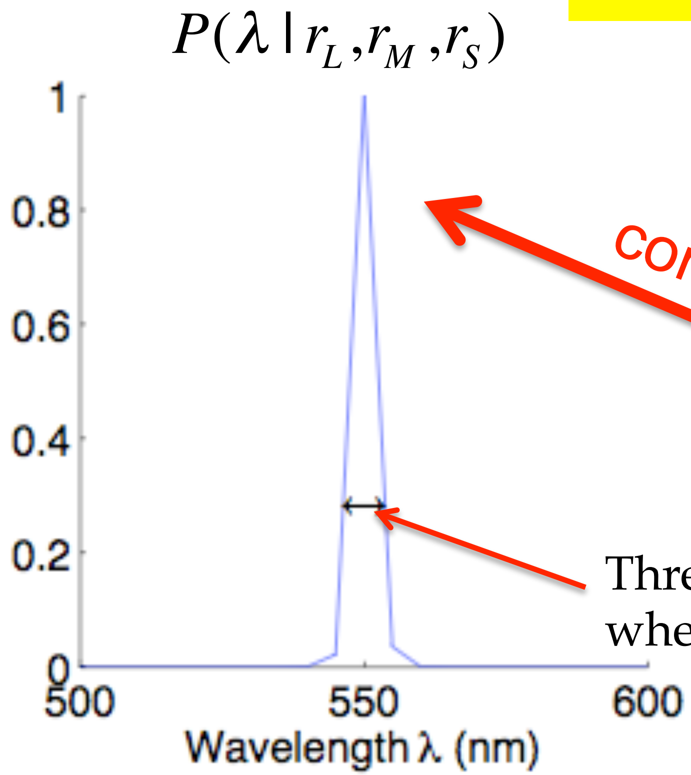
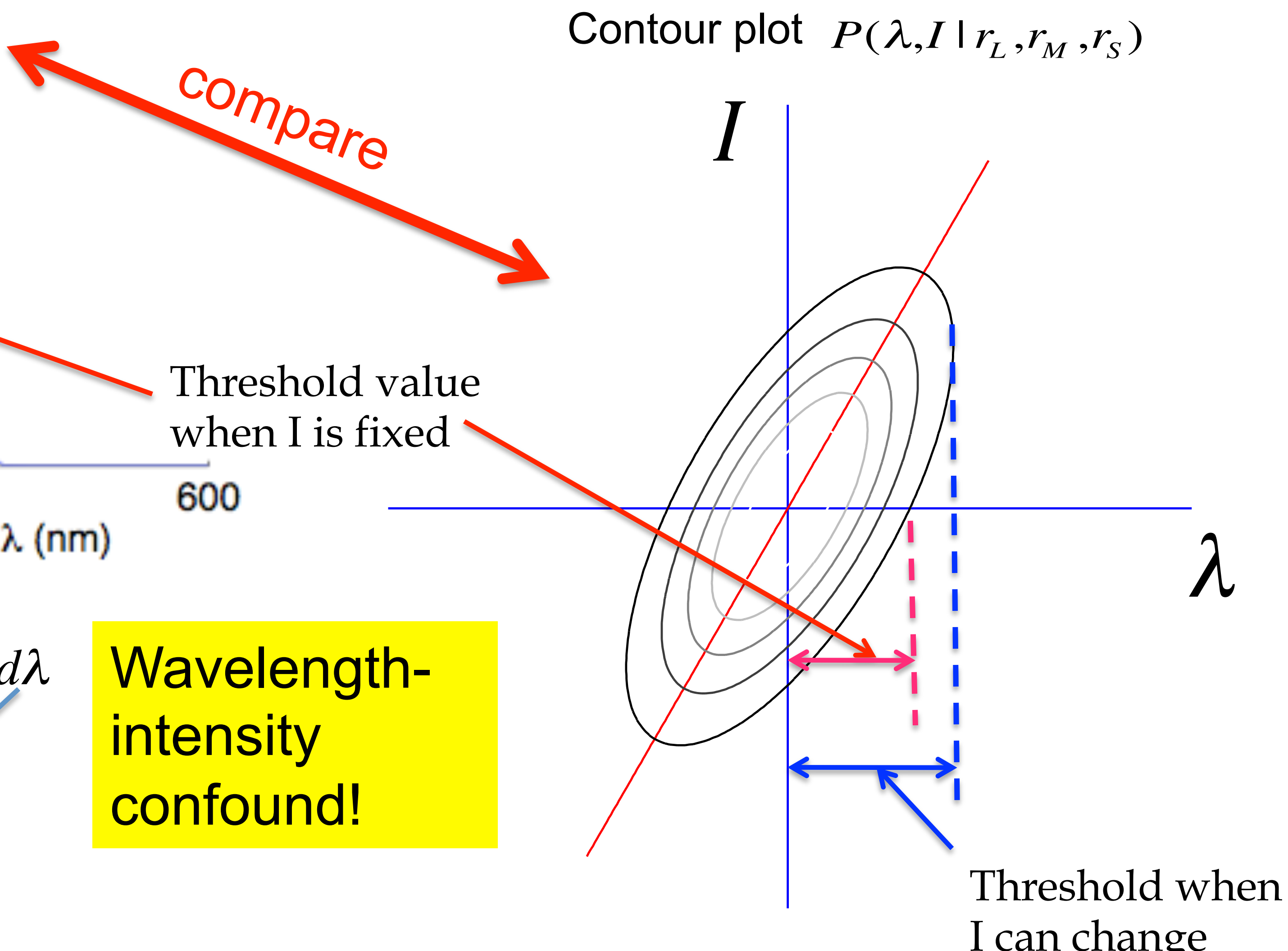
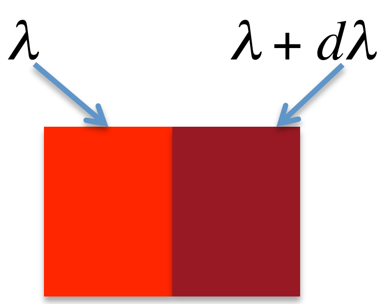
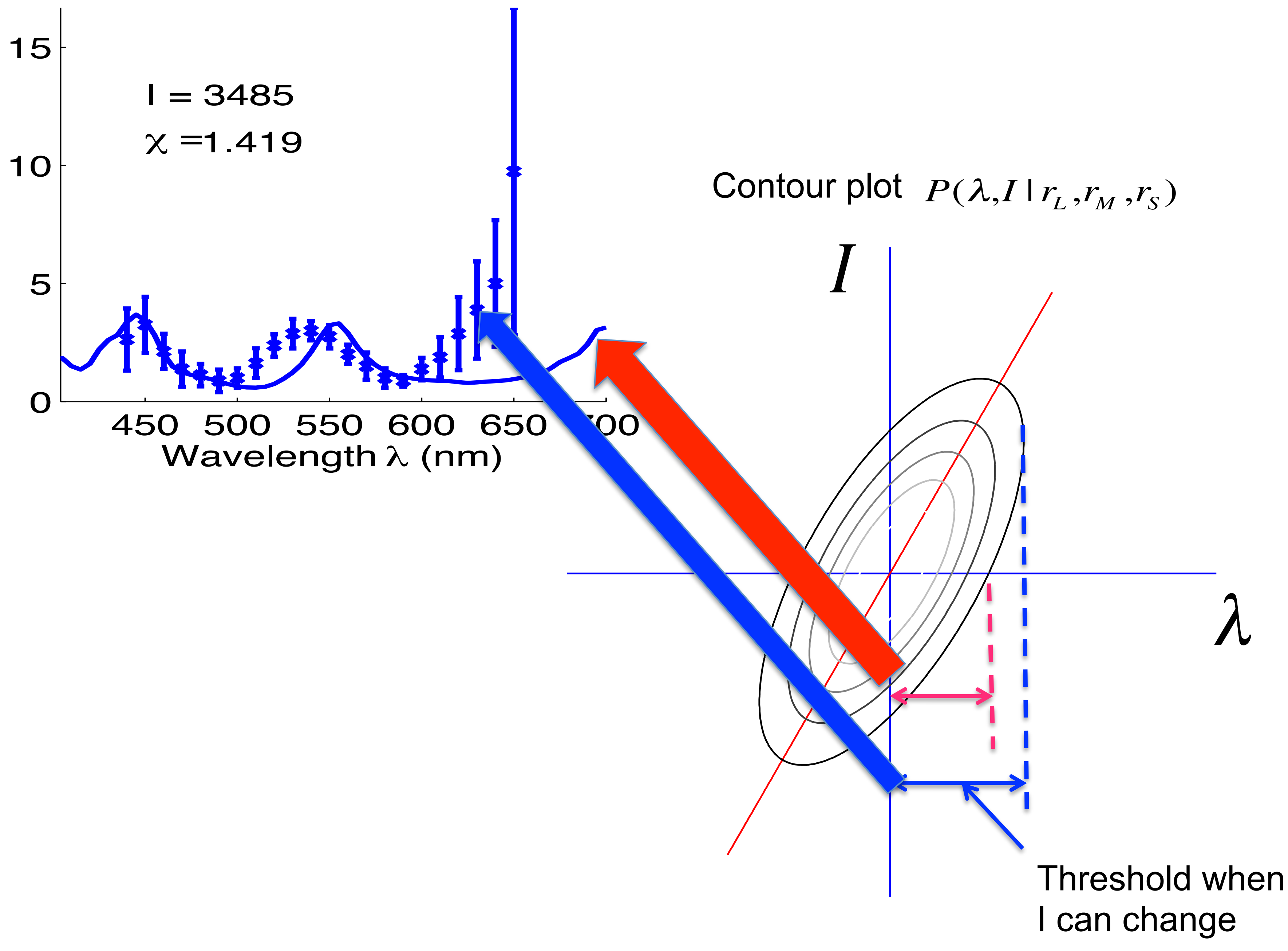






Wavelength-intensity confound!

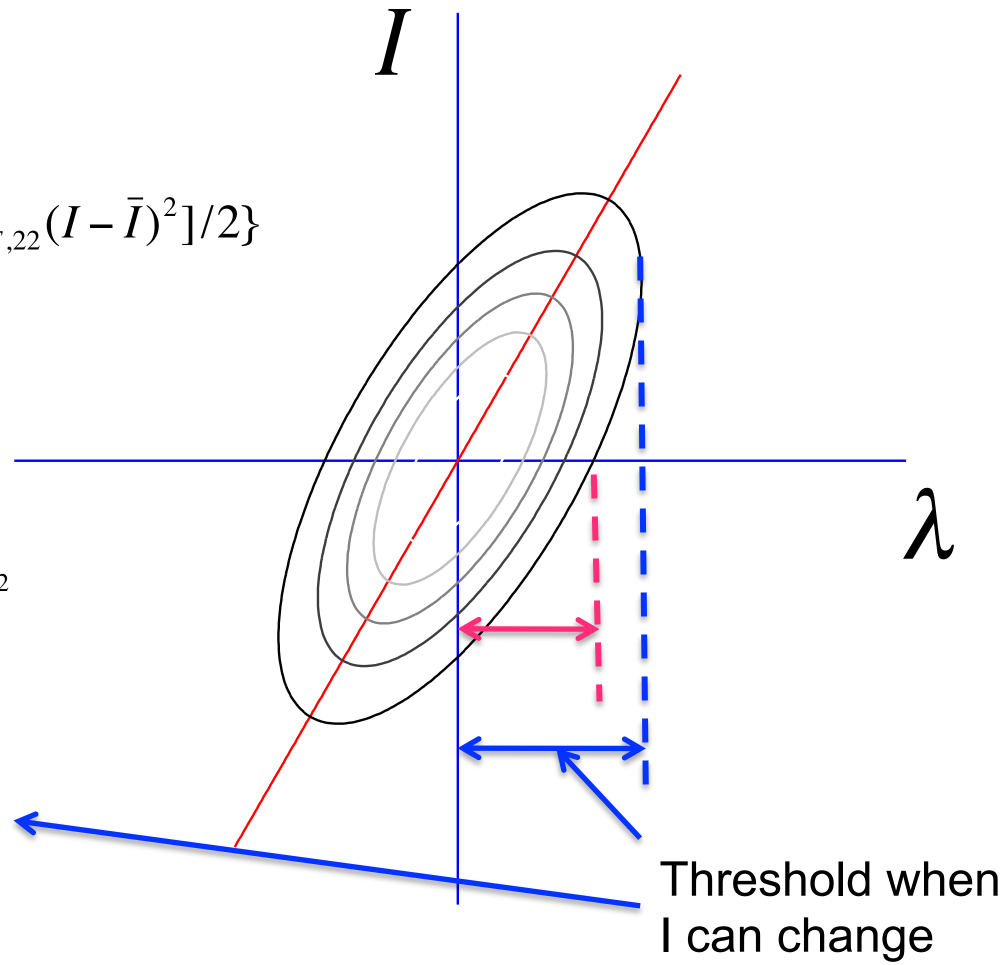
Back to the original measurements (Pokorny and Smith, 1970)



2-d Fisher information formulation

$$I_F(\lambda,I) = -\begin{pmatrix} \left\langle \dfrac{\partial^2 \ln P(r\,|\,\lambda,I)}{\partial\lambda^2} \right\rangle & \left\langle \dfrac{\partial^2 \ln P(r\,|\,\lambda,I)}{\partial\lambda\partial I} \right\rangle \\ \left\langle \dfrac{\partial^2 \ln P(r\,|\,\lambda,I)}{\partial\lambda\partial I} \right\rangle & \left\langle \dfrac{\partial^2 \ln P(r\,|\,\lambda,I)}{\partial I^2} \right\rangle \end{pmatrix}$$

Contour plot $P(\lambda,I\,|\,r_L,r_M,r_S)$

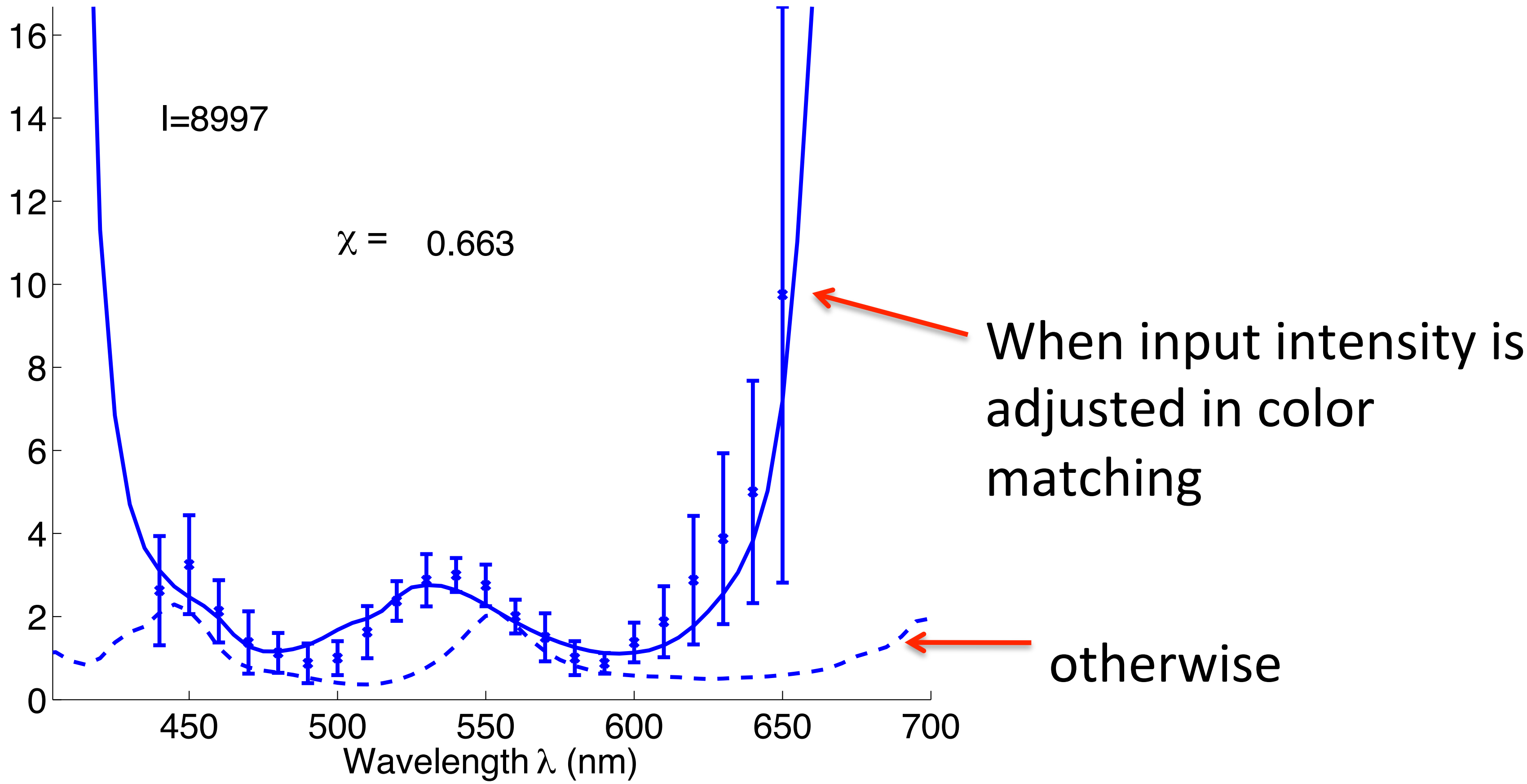


$$P(\lambda,I\,|\,r) \approx$$
$$\exp\{-[I_{F,11}(\lambda-\bar{\lambda})^2 + 2I_{F,12}(\lambda-\bar{\lambda})(I-\bar{I}) + I_{F,22}(I-\bar{I})^2]/2\}$$

Result:

$$\sigma(\lambda) = \frac{1}{\sqrt{I}}\left\{\frac{\sum_a f_a(\lambda)}{\sum_b \frac{[f'_b(\lambda)]^2}{f_b(\lambda)}\sum_c f_c(\lambda) - [\sum_d f'_d(\lambda)]^2}\right\}^{1/2}$$

Better explanation of data!



$$\sigma(\lambda) = \frac{1}{\sqrt{I}}\left\{\frac{\sum_a f_a(\lambda)}{\sum_b \frac{[f'_b(\lambda)]^2}{f_b(\lambda)}\sum_c f_c(\lambda) - [\sum_d f'_d(\lambda)]^2}\right\}^{1/2}$$

Interim Summary:

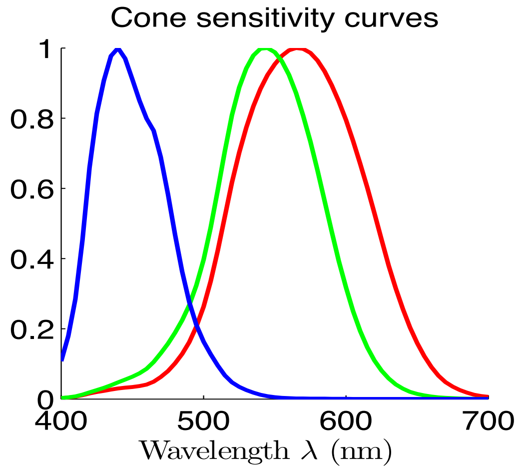
Monochromatic light wavelength discrimination explained by optimal decoding based on signals in the cones.

Suggest that efficiency in information processing efficiency in post-receptoral mechanisms is a constant regardless of wavelength.

Model has to match with experimental methods to account for data.

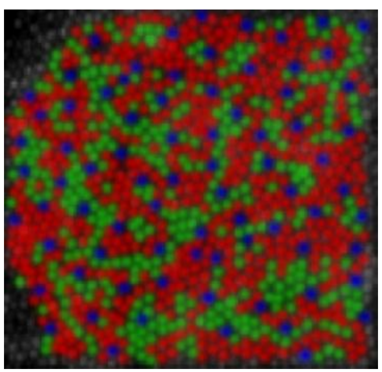
Prediction --- smaller threshold when input intensity is fixed in threshold assessments

The discriminination threshold depends on retinal properties, e.g., cone density effects



$$\sigma(\lambda) = \frac{1}{\sqrt{I}} \left\{ \frac{\sum_a f_a(\lambda)}{\sum_b \frac{[f'_b(\lambda)]^2}{f_b(\lambda)} \sum_c f_c(\lambda) - [\sum_d f'_d(\lambda)]^2} \right\}^{1/2}$$

Number of cones n_L, n_M, n_S



$$n_L f_L(\lambda) \rightarrow f_L(\lambda)$$
$$n_M f_M(\lambda) \rightarrow f_M(\lambda)$$
$$n_S f_S(\lambda) \rightarrow f_S(\lambda)$$

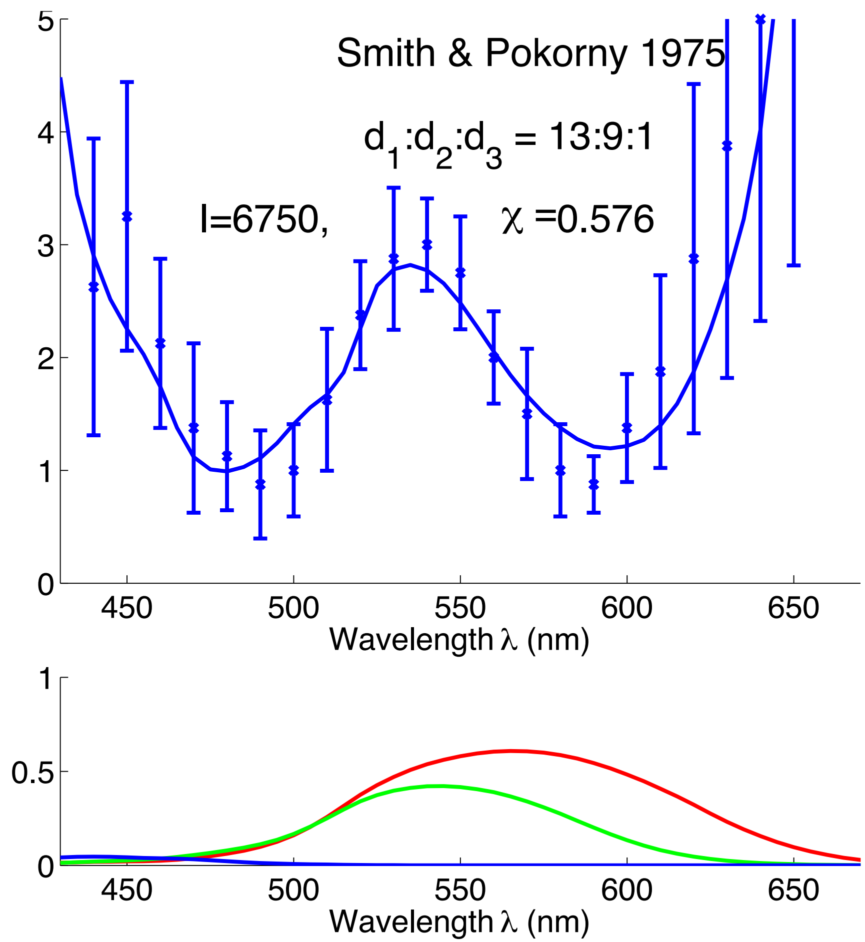
$$n_L : n_M : n_S = 6:3:1$$

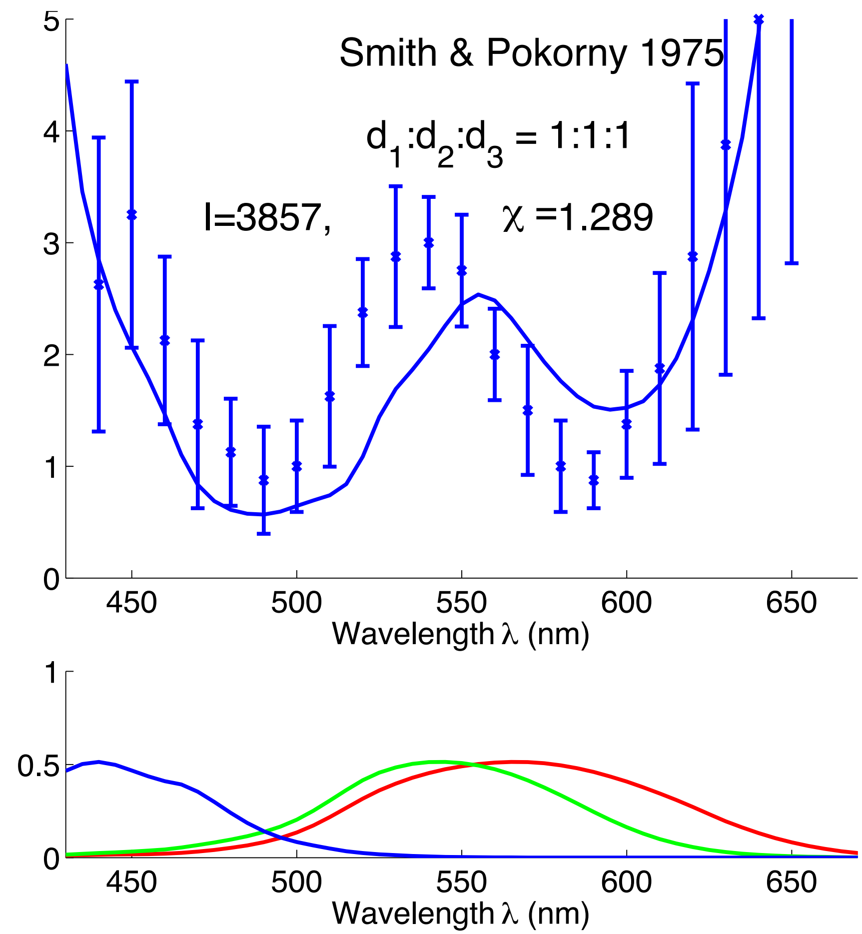
Pre-receptor absorption: O_L, O_M, O_S

$$O_L f_L(\lambda) \rightarrow f_L(\lambda)$$
$$O_M f_M(\lambda) \rightarrow f_M(\lambda)$$
$$O_S f_S(\lambda) \rightarrow f_S(\lambda)$$

$$O_L : O_M : O_S = 1:1:0.2$$

Relative cone densities for L, M, S cones influence model prediction accuracy



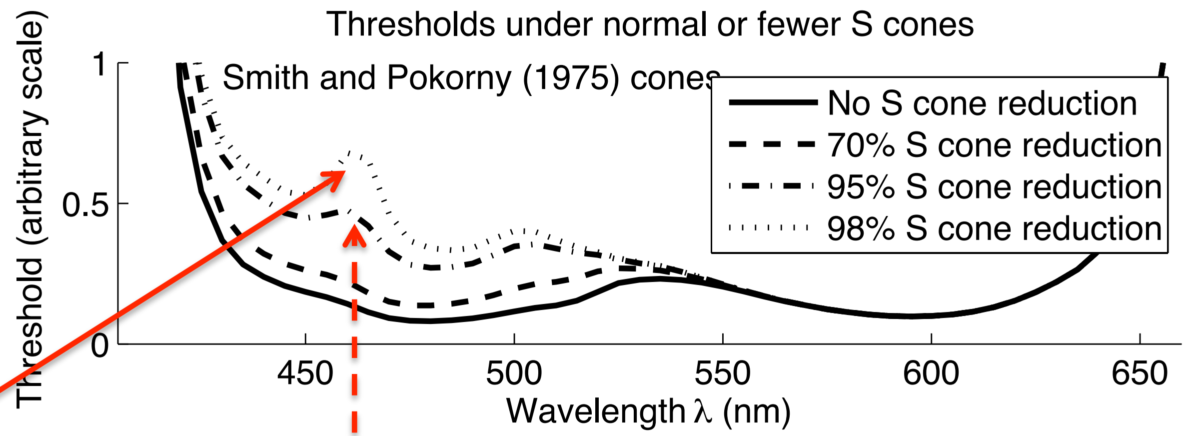


normal amount of S cones

S cones too numerous

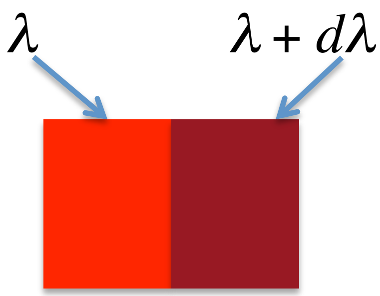
When S cones are too few …

An extra peak… as seen in some data (Bedford and Wyszecki 1958)when the input field is too small, too few S cones



L & M cone co-vary here … becoming color blind.

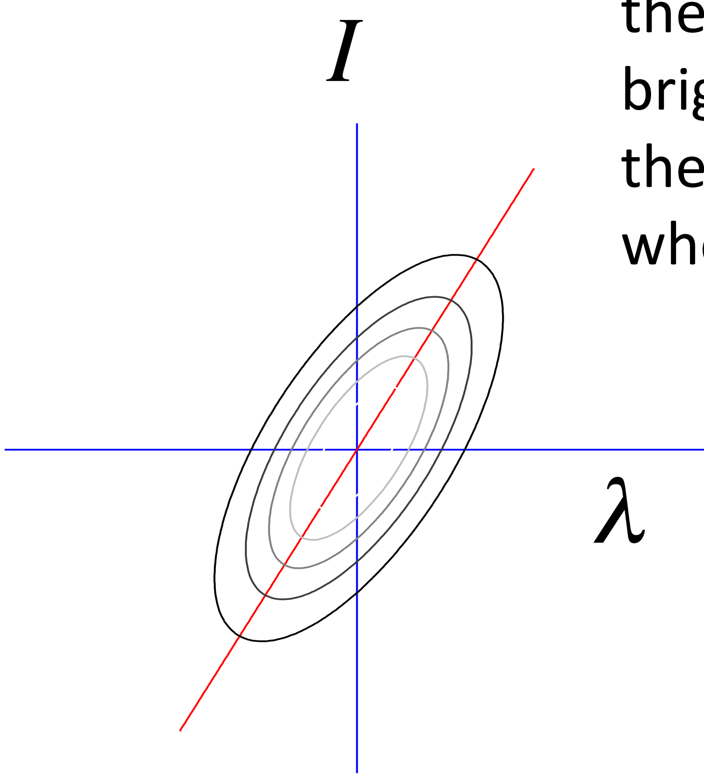
Importance of proper experimental procedures:



Two kinds of procedures in the literature:

Pokorny and Smith (1970): Subject adjust the test field by intensity I to make it appear identical to the standard field, threshold is reached when this is impossible

Bedford and Wyszecki (1958): Subject adjust the test field by intensity I to match the brightness of the two fields, and then see if there is a hue difference. Threshold is reached when there is a hue difference.



Wavelength-intensity confound means that it is difficult to ask subjects to match the brightness of two color fields while checking whether they differ in hue.

Also for dichromats

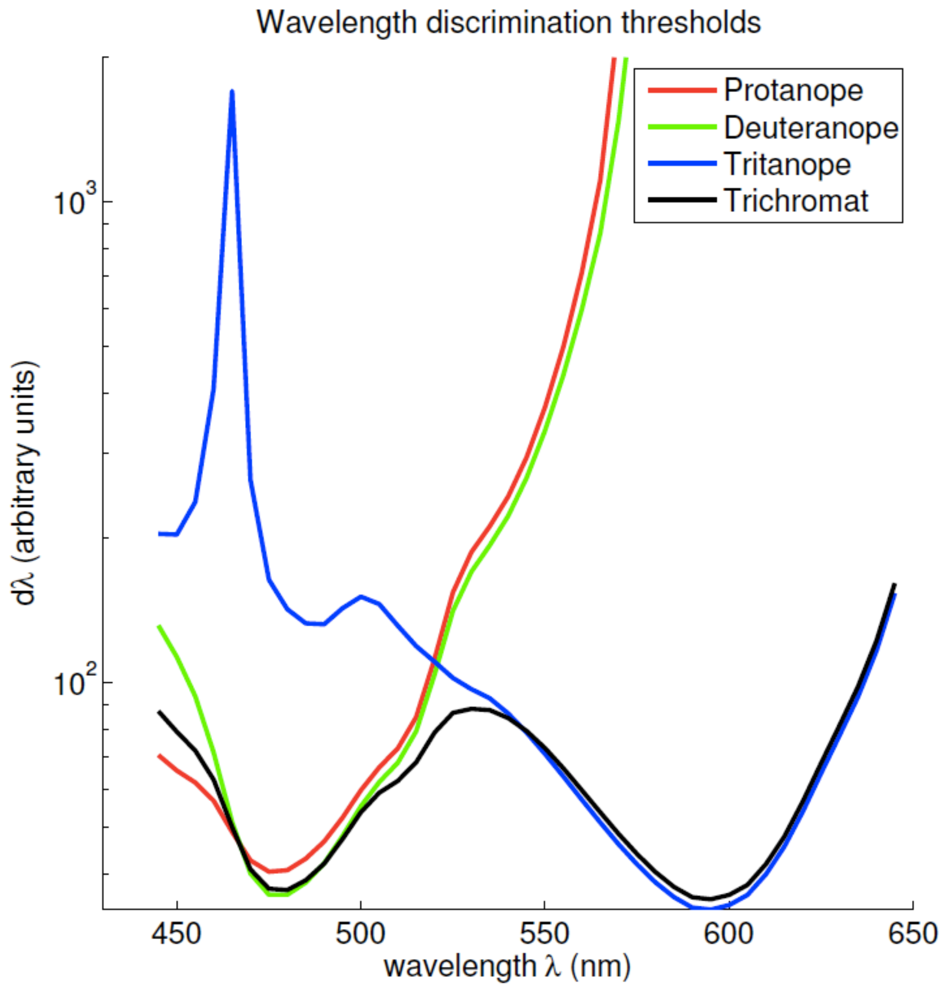


Figure 7: Theoretical preditions of the wavelength discrimination by dichromatics as compared to that by the trichromats. All these curves are by fixing input intensity $I = 1$, while using $f_a(\lambda) = n_a d_a \hat{f}_a(\lambda)$ in which $\hat{f}_a(\lambda)$ is normalized by $\text{Max}_\lambda \hat{f}_a(\lambda) = 1$, while $f_a(\lambda)$ are no longer normalized by $\text{Max}_\lambda \sum_a f_a(\lambda) = 1$. The values $[n_1, n_2, n_3]$ are $[0, 9, 1]$, $[9, 0, 1]$, $[6.7, 3.3, 0]$, and $[6, 3, 1]$ for protanopes, deuteranopes, tritanopes, and trichromats, respectively.

Summary:

Human wavelength discrimination can be understood as optimal decoding from cone absorptions (with constant efficiency)

This model reveals the reliability of data from different experimental procedures.