Homework after lecture 1 of vision course, Jan 2008

1. Let signal S give output O by adding noise N as $O = S + N$, let signal have a gaussian distribution $P(S) = \frac{1}{\sqrt{2\pi}\sigma_s} \exp(-S^2/(2\sigma_s^2))$, and the noise follows an independent gaussian distribution $P(N) = \frac{1}{\sqrt{2\pi}\sigma_n} \exp(-N^2/(2\sigma_n^2))$. Prove that O has a gaussian distribution $P(O) = \frac{1}{\sqrt{2\pi}\sigma_o} \exp(-O^2/(2\sigma_o^2))$ in which $\sigma_o = \sqrt{\sigma_s^2 + \sigma_n^2}$. Please derive the conditional distribution $P(O|S)$ of O given S and the joint distribution $P(O, S)$.

2. Following above, given a gaussian distribution $P(O)$ for O, calculate the entropy: $H(O) = -\int dO P(O)[\log_2 P(O)]$. Calculate the mutual information $I(O; S) = -\int dS dO P(O, S)[\log_2(P(O)P(S)/P(O, S))]$ and please convince yourself that the results are the same as the formula given in the lecture.

Please complete these homeworks before the next lecture for best effectiveness in following the lectures.